EKONOMI PRODUKSI

Kode PTE-4103

PERTEMUAN KESEBELAS:

Maximization in a Two-Output Setting

11

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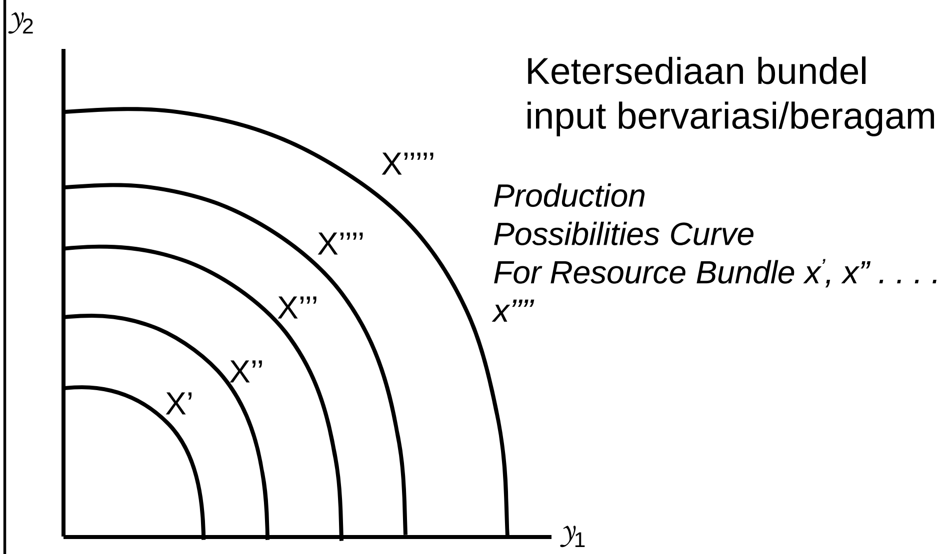
Sub-Pokok Bahasan:

1. The Family of Product Transformation Function
2. Maximization of Output
3. The Isorevenue Line
4. Constrained Revenue Maximization
5. Simple Mathematics of Constrained Revenue Maximization
6. Minimization of Input Use Subject to a Revenue Maximization

Sumber Bacaan:

Debertin. 1986. *Agricultural Production Economic*. Macmillan. New York: Chapter 16

1. The Family of Product Transformation Function



- *The Family of Product Transformation Function* seperti *family isoquant*

- Dua fungsi transformasi produk tidak saling bersentuhan atau berpotongan dg yg lain

- Masing^2 fungsi transformasi produk berurutan diasumsikan dg tingkat penggunaan bundel input yg berbeda

2. Maximization of Output

Asumsi:

tdk terdpt keterbatasan bundel input yg tersedia → pers fungsi transformasi produk adalah

$$x = g(y_1, y_2)$$

Keputusan petani:

Berkeinginan menetapkan kuantitas input x yg diperlukan untuk output y_1 & y_2 maksimum

Turunan fungsi transformasi produk

dx/dy_1 dan dx/dy_2

dx/dy_1 adalah $1/(dy_1/dx)$ atau $1/MPP_{xy_1}$

$dx/dy2$ adalah $1/(dy2/dx)$ atau $1/MPP_{xy_2}$

Menjelaskan tambahan biaya dr tambahan memproduksi unit y_1 & y_2 yg diekspresikan dlm bentuk kuantitas bundel input secara fisik

Jika jumlah kedua output pd global optimum, tambahan satu unit bundel input tdk akan menambah output y_1 maupun y_2.

→ Tambahan produk (*product marginal*) dr x untuk produksi y_1 (MPP_{xy_1}) & untuk produksi y_2 (MPP_{xy_2}) akan nol

3. The Isorevenue Line

Fungsi penerimaan dr petani yg memproduksi dua output

$$R = p_1 y_1 + p_2 y_2$$

Asumsi petani memerlukan penerimaan sebesar $ 1 000

p_1 = $ 5 & p_2 = 2 $

Petani bisa memilih u/ memproduksi semua

y_1 (200 = $ 1 000/$ 5) atau

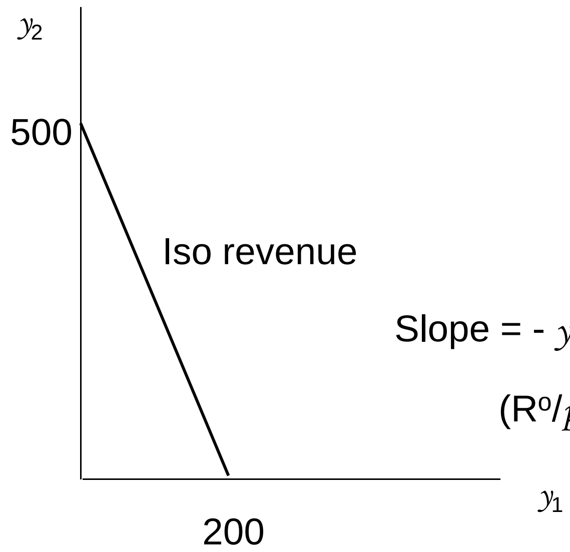
y_2 (500 = $ 1 000/$ 2)

kombinasi diantara keduanya → tabel

Kombinasi	Unit y_1	Unit y_2	Revenue
A	200	0	1 000
B	150	125	1 000
C	100	250	1 000
D	50	375	1 000
E	0	500	1 000

Slope isorevenue

- Bila semua berasal dari penjualan semua output y_1 → $y_1 = R^o/p_1 = 200$
- Bila semua berasal dari penjualan semua output y_2 → $y_2 = R^o/p_2 = 500$

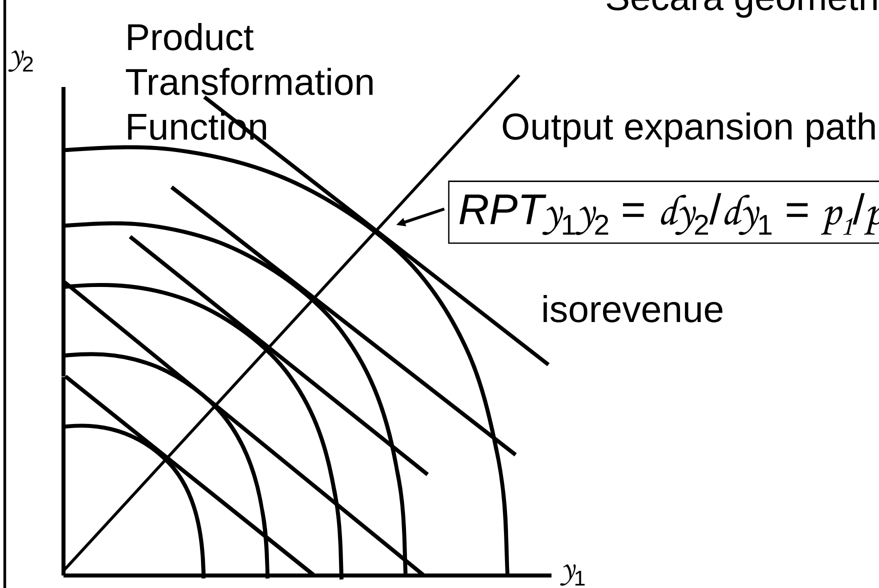


Slope = $-\ y_2/y_1$

$(R^o/p_2)/(R^o/p_1) = -\ p_1/p_2$

4. Constrained Revenue Maximization

Secara geometri



$RPT_{y_1 y_2} = dy_2/dy_1 = p_1/p_2$

$$- RPT\ y_1y_2 = - dy_2/dy_1 = (1/MPPx_{y1})/(1/MPPx_{y2})$$

$$= MPPx_{y1}/MPPx_{y2}$$

Keduanya negatif $= - p_1/p_2$

$$\boxed{RPT y_1y_2 = dy_2/dy_1 = p_1/p_2}$$

$p_1/p_2 = 5/2 = 2.5$

X to *Corn*	Y_1 (bu/acre)	*MPPx in Corn*	X to *Soybean*	Y_2 (bu/acre)	*MPPx in Soybean*	*RPT* of Corn for Soybean
0	0		10	55		
1	45	45	9	54	55-54=1	1/45=0.022
2	62	17	8	52	2	2/17=0.118
3	87	15	7	49	3	3/15=0.200
4	100	13	6	45	4	0.305
5	111	11	5	40	5	0.455
6	120	9	4	34	6	0.667
7	127	7	3	27	7	1.00
8	132	5	2	19	8	1.60
9	135	3*)	1	10	9	3.00
10	136	1	0	0	10	10.00

5. Simple Mathematics of Constrained Revenue Maximization

5.a. Model I

Fungsi tujuan: max. revenue

$$\text{Max } p_1 y_1 + p_2 y_2$$

Kendala: bundel input yg tersedia

$$x^o = g(y_1, y_2)$$

Pers Lagrang

$$L = p_1 y_1 + p_2 y_2 + \theta\,[x^o - g(y_1, y_2)]$$

FOC:

$$\left.\begin{aligned} \partial L/\partial y_1 &= p_1 - \theta\, \partial g/\partial y_1 \\ \partial L/\partial y_2 &= p_2 - \theta\, \partial g/\partial y_2 \end{aligned}\right\} \quad p_1/p_2 = (\partial g/\partial y_1)/\,(\partial g/\partial y_2)$$

$$\partial L/\partial \theta = x^o - g\,(y_1, y_2)$$

Selama g adalah x

$$p_1/p_2 = (1/\text{MPP}_{xy_1})/(1/\text{MPP}_{x_y 2})$$

$$- \text{MPP}_{xy_2}/\text{MPP}_{x_y 1}) = - p_1/p_2$$

$$\boxed{RPT_{y_1 y_2} = p_1/p_2}$$

Slope fungsi transformasi produksi = slope isorevenue

Tahapan lain:

$p_1 - \theta\, \partial g/\partial y_1 = 0 \rightarrow \theta = p_1/(\partial g/\partial y_1)$

$p_2 - \theta\, \partial g/\partial y_2 = 0 \rightarrow \theta = p_2/(\partial g/\partial y_2)$

$\Rightarrow$

$p_1/(\partial g/\partial y_1) = \theta$

$p_2/(\partial g/\partial y_2) = \theta$

$\Downarrow$

$$p_1/(\partial g/\partial y_1) = p_2/(\partial g/\partial y_2) = \theta$$

$$p_1 MPPx y_1 = p_2 MPPx y_2 = \theta$$

Prinsip *the equimarginal return* (tambahan penerimaan yg sama) $\rightarrow$ $VMPx y_1 = VMPx y_2 = \theta$

5.b. Model II

Fungsi tujuan: max. revenue

$$Max\;\; p_1 y_1 + p_2 y_2$$

Kendala: anggaran (*buged*)

$$C^o = vx^o = vg(y_1, y_2)$$

Pers Lagrang

$$L = p_1 y_1 + p_2 y_2 + \phi[C^o - vg(y_1, y_2)]$$

FOC: $\partial L/\partial y_1 = p_1 - \phi v \partial g/\partial y_1 = 0$

$\partial L/\partial y_2 = p_2 - \phi v \partial g/\partial y_2 = 0$

$\partial L/\partial \phi = C^o - vg(y_1, y_2) = 0$

Pembagian p

$p_1/p_2 = (\partial g/\partial y_1)/(\partial g/\partial y_2)$

$\boxed{RPT_{y_1 y_2} = p_1/p_2}$

Slope fs transformasi produk

Slope isorevenue

Tahapan lain:

$p_1 - \phi v \partial g/\partial y_1 = 0 \rightarrow p_1/v(\partial g/\partial y_1) = \phi$

$p_2 - \phi v \partial g/\partial y_2 = 0 \rightarrow p_2/v(\partial g/\partial y_2) = \phi$

$p_1/v(\partial g/\partial y_1) = p_2/v(\partial g/\partial y_2) = \phi$

$\boxed{VMPx_{y_1}/v = VMPx_{y_2}/v = \phi}$

Petani dpt mengalokasikan bundel input dlm situasi pengeluaran nilai uang terakhir pd bundel input yang menghasilkan rasio yg sama antara VMP & biaya pd kedua output → tambahan nilai produksi krn penambahan satu unit input = biaya per unit input.

Mahasiswa dipersilahkan mempelajari sendiri contoh penyelesaian alokasi input u/ 2 output dg fungsi produksi berikut:

$$y_1 = x^{0.33}y_1$$

$$y_2 = x^{0.5}y_2$$

Total input yg tersedia:

$$x = x_{y1} + x_{y2}$$

6. Minimization of Input Use Subject to a Revenue Maximization

Fungsi tujuan: min. bundel input x

$$Min \;\; g(y_1, y_2)$$

Kendala: penerimaan

$$R^o = p_1 y_1 + p_2 y_2$$

Pers Lagrang

$$\mathcal{L} = g(y_1, y_2) + \psi(R^o - p_1 y_1 - p_2 y_2)$$

FOC: $\partial L/\partial y_1 = g_1 - \psi p_1 = 0 \Rightarrow g_1 = \psi p_1$

$\partial L/\partial y_2 = g_2 - \psi p_2 = 0 \Rightarrow g_2 = \psi p_2$

$\partial L/\partial \psi = \mathcal{R}^o - p_1 y_1 - p_2 y_2) = 0$

Pembagian g

$$g_1/g_2 = (\partial g/\partial y_1)/(\partial g/\partial y_2) = (1/MPP_{xy_1})/(1/MPP_{xy_2})$$

$$= MPP_{xy_2}/MPP_{xy_1}$$

$$= RPT_{y_1 y_2} = dy_2/dy_1 = p_1/p_2$$

$\partial L/\partial y_1 = g_1 - \psi p_1 = 0 \Rightarrow g_1/p_1 = \psi$

$\partial L/\partial y_2 = g_2 - \psi p_2 = 0 \Rightarrow g_2/p_2 = \psi$

$$\boxed{g_1/p_1 = g_2/p_2 = \psi}$$

→ VMPg1 (∂g/∂y1) =

$g_1 = (\partial g/\partial y_1) = (1/MPP_{xy_1}) \rightarrow MPP_{xy_1} = 1/g_1$

$g_2 = (\partial g/\partial y_2) = (1/MPP_{xy_2}) \rightarrow MPP_{xy_2} = 1/g_2$

$VMP_{xy_1} = p_1 \cdot MPP_{xy_1}$

$VMP_{xy_2} = p_2 \cdot MPP_{xy_2}$

$MPP_{xy_1} = 1/g_1$

$MPP_{xy_2} = 1/g_2$

$VMP_{xy_1} = p_1 \cdot MPP_{xy_1}$

$VMP_{xy_2} = p_2 \cdot MPP_{xy_2}$

$VMP_{xy_1} = p_1 \cdot 1/g_1$

$VMP_{xy_2} = p_2 \cdot 1/g_2$

$\Downarrow$

$g_1/p_1 = 1/VMP_{xy_1}$

$g_2/p_2 = 1/VMP_{xy_2}$

$\boxed{g_1/p_1 = g_2/p_2 = \psi} \Rightarrow \boxed{1/VMP_{xy_1} = 1/VMP_{xy_2} = \psi}$

Referensi

Debertin.1986. Agricultural Production Economics. Macmillan. New York: chapter 16